

Investigating vibro-driven monopile installation into sand in a geotechnical centrifuge

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ABSTRACT

Bottom-fixed foundations currently dominate the offshore wind market. The installation of monopiles for offshore wind turbines by vibration is a promising alternative to the conventional impact-driven technique. Despite being a low-noise, time-efficient and cost-effective alternative, take-up of this method has been slow because uncertainties remain on the pile drivability and post-installation performance of vibro-installed monopiles, which translates to risk for the industry. Carrying out real scale field tests to increase understanding is extremely expensive and time-consuming. In contrast, centrifuge model tests are an alternative for investigating the impact of vibro-installation on the post-installation performance of monopiles for different soil conditions. The paper presents the development and first use of a mini vibro-driver in a geotechnical centrifuge. It highlights the importance of performing tests in such a way as to appropriately replicate soil mechanisms during vibro-driving. Finally, results from an initial pile installation test in the centrifuge are presented.

Keywords: monopiles, vibro-driving, physical modelling, geotechnical centrifuge

1 INTRODUCTION

Monopiles are the preferred foundation solution for offshore wind turbines (OWTs). The use of vibro-drivers for the installation of monopiles is potentially more efficient than the conventional impact-hammering technique, resulting in reductions in total installation times, steel fatigue damage during installation and environmental noise levels. However, there is a lack of understanding on the following key elements of vibro-driving:

- pile drivability: the choice of a set of vibro-parameters to install the monopile in a given soil profile and prediction of the installation rate;
- pile post-installation performance: lateral and vertical capacities and stiffnesses cannot be assessed easily and with confidence.

Van Dorp et al. (2019) highlighted that the database

available to calibrate prediction methods for vibro-installed monopiles is extremely limited because pile driving monitoring is not common practice. Holeyman and Whenham (2017) also stated that, because of the lack of broad-based correlation data, a vibratory driving prediction method fully recognised by the profession does not exist. Achmus et al. (2020) concluded from real scale field tests in sand that monopile performance is sensitive to vibro-installation parameters and further research is needed to identify the optimum set of parameters to maximise the bearing behaviour.

In summary, it is clear that additional information is required to understand the processes occurring during vibro-driving and the effects these have on post-installation response. Although attempts have been made to investigate the problem numerically (Machacek et al., 2021; Daryaei et al., 2020; Staubach et al., 2020),

given the number of competing mechanisms occurring during vibrodriving and the consequent challenges of capturing the constitutive response of the different soil elements in the system, it is believed that field testing and physical modelling are likely to lead to the best insights initially.

Data from large diameter monopiles are scarce (Achmus et al., 2020; LeBlanc, 2014; Neef et al., 2013). Carrying out real scale field tests to increase understanding is extremely expensive and time-consuming and there always remains uncertainty in the geotechnical conditions at the selected field site (requiring an extensive site investigation and set of follow-on element testing). Small-scale (laboratory) tests offer a cost-effective option, allowing physical evidence to be obtained under controlled and repeatable conditions.

Despite cost and schedule advantages, when conducting small-scale testing, careful consideration needs to be taken of scale effects to ensure that the information provided from the tests is relevant. Consequently, the next section will describe some challenges with physical modelling to ensure that the soil mechanisms occurring during vibro-driving are replicated. In particular, the importance of representing field stresses through the enhanced gravity field generated in a geotechnical centrifuge is discussed.

2 MECHANISMS TO CONSIDER DURING PHYSICAL MODELLING OF VIBROPILING USING A GEOTECHNICAL CENTRIFUGE

Small scale tests carried out in laboratory conditions without any stress enhancement in the soil are usually suitable to capture only qualitative aspects of soil mechanisms. This is because the stress levels in the model will be $1/N$ times that in the field, where N is the model (length) scale, and the mechanical response of the soil at these stresses will vary from those in the field.

Geotechnical centrifuge modelling allows more reliable quantitative investigations. Geotechnical centrifuges were created so the geometrical size of the prototype could be reduced while still replicating stress conditions in field, by imposing an enhanced acceleration gravity field on the soil. By spinning a soil sample, a centrifuge increases the centrifugal acceleration and the stress levels within the soil. For example, if the model is geometrically scaled down by a factor N of 100, the induced acceleration must be 100 times Earth's gravity (100g) to maintain similitude.

Axial soil resistance during driving or axial loading can be divided into two components: shaft and tip resistance. Vibro-driving is characterized by a dramatic reduction of shear stresses at the pile-soil interface during installation. Van Dorp et al. (2019) noted that, for granular soils, the soil rapidly regains its strength after the termination of vibro-installation, possibly even exceeding its original strength due to densification. The

authors quoted results obtained during the Riffgat Project (Neef et al., 2013), where vibro-installation of monopiles was followed by impact-hammering. The high blow count indicated axial capacity recovery after the termination of vibro-installation. However, the degree of recovery following vibro-installation, in comparison to original in situ conditions and those pertaining after impact-driving piles, needs to be quantified better, not only in terms of magnitude but also the mechanisms for such recovery.

In the following sub-sections, the shaft and tip resistance of vibrated piles are discussed. The importance of using centrifuge tests to properly replicate mechanisms during vibro-installation are highlighted.

2.1 Shaft resistance

The three main possible causes discussed in the literature for the reduction in shaft friction for vibrated-piles are: friction fatigue, cyclic acceleration and induced excess pore pressures.

The term *friction fatigue* (or stress relaxation) refers to the decrease of the ultimate shaft friction mobilized in the pile-soil interface as the pile penetrates further (White & Lehane, 2004). Because this phenomenon is controlled mainly by the cyclic history during the pile installation, it is very natural to try to relate it to the decrease in shaft friction during vibro-installation. The effect of friction fatigue has been studied for different installation methods, but there is a paucity of quantitative data for vibro-installation.

Vogelsang et al. (2015) observed that for monotonic, quasi-static and vibratory penetration in 1g tests, soil adjacent to the pile experiences slight uplift when the pile tip has passed, although for both quasi-static and vibro-tests this is followed by a clear trend of soil moving towards the pile. The authors associate this trend with friction fatigue and to similar trajectories described by White and Bolton (2004).

Moriyasu et al. (2018) showed that large shear stresses were mobilized near the pile tip during vibro-installation, with the shear stress decreasing with increasing distance from the pile tip – a characteristic of friction fatigue. They noted that vibratory driving conditions such as frequency and vibratory driving forces may affect the friction fatigue, and that the shaft friction normalized by the cone tip resistance seems to converge to a residual value over the accumulated shear work performed by vibratory driving. However, the data are insufficient to quantify these effects with confidence.

As friction fatigue is related to stress relaxation in the vicinity of the pile, it is reasonable to believe that the phenomenon is highly dependent on soil strength and stiffness parameters – at least from a quantitative point of view. For this reason, the use of centrifuge tests to properly match the stress conditions in the soil is critical to assess friction fatigue effects in vibro-driving.

In addition, it is not yet clear if the high number of cycles imposed on the pile would solely be enough to

generate the significant decrease of shear stresses on the pile shaft, or if there are also dynamic effects associated with vibro-installation. The cyclic acceleration theory offers an interesting perspective.

Cyclic acceleration motion of the soil grains was highlighted by Rodger & Littlejohn (1980) and Viking (2002) as a key mechanism behind shear stress reduction during vibro-piling. As described by Viking (2002), as the pile firstly experiences an upward movement, the soil grains at the pile surface tend to follow its movement. This creates upward momentum of the soil grains. As the pile reverses its movement, the grains cannot instantaneously follow the downward cycle due to their upward momentum and the high downward acceleration of the pile. The pile-soil relative movement therefore becomes out of phase. During this reversal the individual soil grains experience a ‘free-fall phase’, with short-term drops in intergranular contact, causing the vertical confining stress to reduce significantly. This phenomenon is believed to occur when the pile undergoes accelerations in excess of Earth’s gravity (or the static acceleration field in a centrifuge).

In the absence of such dynamic effects, there will be no difference between vibro-driving and quasi-static cyclic jacking, other than the much larger numbers of cycles for the former. It is therefore important to correctly scale the properties affecting the dynamic response, such that the resulting soil-structure-interaction represents the conditions intended to be investigated (and comparisons can potentially be made with quasi-static cyclic jacking). In the design of the centrifuge test apparatus, this was achieved by ensuring that the pile maximal vertical acceleration exceeded the background centrifugal acceleration.

Viking (2002) highlighted that **induced excess pore pressure** also plays an important role in the reduction of the soil shear strength during vibro-pile installation in saturated soils. Doherty et al. (2015) noted that, during the dynamic process of vibro-installation in sand, there is rapid development of excess pore pressures around the pile, which are not able to dissipate during installation and affect the whole installation behaviour. Once vibro-installation is completed, the excess pore pressures would dissipate, and the soil elements would experience small volume reductions. However, further investigation is necessary to understand the excess pore pressure mechanisms during vibro-driving. It is also important to highlight that vibro-installation is effective not only in saturated sand, but also in dry sand. For this reason, mechanisms other than induced excess pore pressure generation must be reducing shaft shear stresses.

In addition to the above cited mechanisms, the overall shaft resistance of piles is clearly related to the development of interface shear. This will depend on the absolute interface **roughness** (R_a). For the present model, the field interface roughness was replicated in order to correctly model the residual interface friction

coefficient. This may introduce scaling issues associated with shear band scaling (e.g. Garnier et al., 2007).

There is no consensus on the relative contributions of these mechanisms on shaft friction reduction during vibro-driving. The present research aims to investigate the various influencing factors systematically to provide evidence-based guidance for vibro-driving monopiles in sand.

2.2 Tip resistance

Rodger and Littlejohn (1980) proposed two types of vibratory driving motion, each presenting a different tip mechanism: fast and slow vibratory driving. In the original study, the authors clearly state that it is the relationship between the soil density in situ and the critical soil density that establishes whether slow or fast vibratory driving will occur. However, in more recent publications (Dierssen, 1994; Vogelsang, 2016; Labenski and Moormann, 2019; Massarsch et al., 2020), the difference between the two vibro-driving motion lies specifically in the tip behaviour. The two different tip mechanisms – explained in the following paragraphs – seem to be well-accepted as a fundamental behaviour for vibro-driving.

Slow vibratory driving is considered to be the dominant case in vibro-installation (Massarsch et al., 2020). In this case, full reversal of motion occurs, the soil under the tip is completely unloaded and the pile tip intermittently loses contact with the underlying soil. This process is illustrated in Figure 1. Firstly, the pile is just starting its upward cyclic movement, and the soil is unloaded (1-2). The pile tip loses contact with the underlying soil and as it moves upward (2-3). Then, the pile reverses its movement (3-4) and during its downward movement, regains contact with the soil and reloads it (4-1’). Penetration is assumed to occur in an elasto-plastic behaviour (Rodger and Littlejohn, 1980).

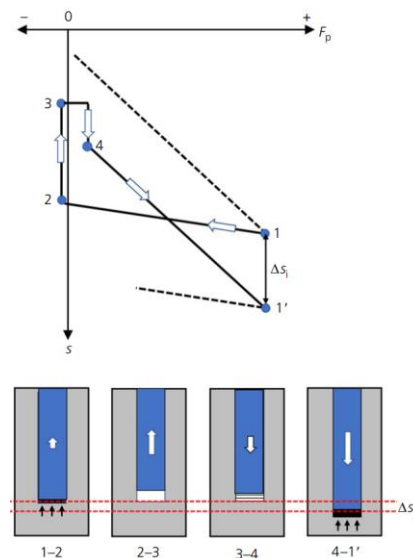


Fig. 1. Simplified model of slow vibratory motion, where F_p is the force in the pile and Δs_i the pile toe displacement (Massarsch et al., 2020, modified from Dierssen (1994)).

During fast vibratory driving reversal of motion also occurs, but the pile tip is not completely unloaded, and it does not lose contact with the underlying soil. Thus, in this mechanism the pile tip will behave more similarly to a monotonically jacked pile (Hoffmann et al., 2020). Penetration is assumed to occur mainly in plastic behaviour (Labenski and Moormann, 2019).

Massarsch et al. (2020) and Labenski and Moormann (2019) stated that the cyclic amplitude of the pile is an important factor on the final vibration mode. If the cyclic amplitude is too small, the pile tip resistance is not completely unloaded and the tip will not lose contact with the underlying soil.

For this research, the cyclic amplitude of the vibrations was a result of the properties of the vibro-driver. These are discussed further below and are expected to generate condition similar to those occurring during field events.

3 PRELIMINARY CENTRIFUGE TESTING

3.1. The model vibro-driver

An innovative miniature vibro-driver for centrifuge environment was developed at UWA with further details provided in Mazutti et al. (2022). It was found that a device using counter-rotating masses (as used in field vibro-drivers, e.g. Neef et al. (2013)) was impractical. This was because for scaling reasons the device had to be small (ideally $1/N$ times that of the field device), of low mass (ideally $1/N^3$ of the prototype), and apply a dynamic force $1/N^2$ of the prototype at a higher frequency than in the field. For a system involving counter-rotating masses, this would have required impractically fast motor rotation rates, very small mass eccentricities and difficulties in synchronisation.

Instead, a device was developed whereby a driving wheel with seven undulations (driven by a DC motor) drove a 'slider' mass up and down. The advantage of this 'bumpy' wheel is that the rotation rate required for the motor was seven times slower than the frequency of the delivered dynamic force and the dynamic force could be controlled by selection of the amplitude of the undulations on the wheel, the mass of the slider and the rotation rate of the motor.

Following design and fabrication, the model vibro-driver was first characterised through centrifuge N_g and $1g$ (i.e. non-spinning) tests where the device was mounted on a load cell, attached directly to the centrifuge strongbox. This allowed direct measurement of the dynamic load delivered which the device shown to be able to produce approximately sinusoidal variations of force of frequencies up to approximately 400 Hz for centrifuge accelerations of up to 80g.

3.2 Test conditions for pile installation test

The initial pile tests were performed in the large beam centrifuge (with an effective radius of 5 m) at the National Geotechnical Centrifuge Facility (NGCF) at the

University of Western Australia (UWA). A model scale of 30 was used, so the centrifuge was spun at 30g.

The test was carried out in UWA superfine silica sand with a mean particle size $d_{50} = 0.18$ mm, maximum and minimum dry unit weights of 17.7 and 14.7 kN/m³, and coefficient of uniformity $C_u = 1.9$ (Chow et al., 2019).

The mini vibro-driver was mounted on the top of a model monopile, which has external diameter, $D = 50$ mm, length, $L = 500$ mm and wall thickness, $t = 0.5$ mm. At 30g for this preliminary test represents a pile with $D = 1.5$ m, $L = 15$ m (somewhat smaller than current monopiles) and $D/t = 100$ (typical for monopiles). The vibro-driver and pile system had a total mass of 1.51 kg (representing a system of 41 tonnes at 30g). Future testing is expected to be conducted at larger centrifuge accelerations to model larger piles and to explore the effect of scale.

An accelerometer was attached near the top of the pile to measure acceleration during installation and was logged continuously at 20 kHz. A rod at the top part of the vibro-driver, sliding through a linear rail, ensured verticality of the pile during installation. The overall vertical movement of the pile during installation was measured by a laser displacement transducer mounted on an actuator positioned above the rod. Due to limitations on the measurement rate of the displacement transducer, the vertical pile oscillations within each cycle could not be measured directly but were deduced from the pile accelerometer readings. The setup is shown in Figure 2.

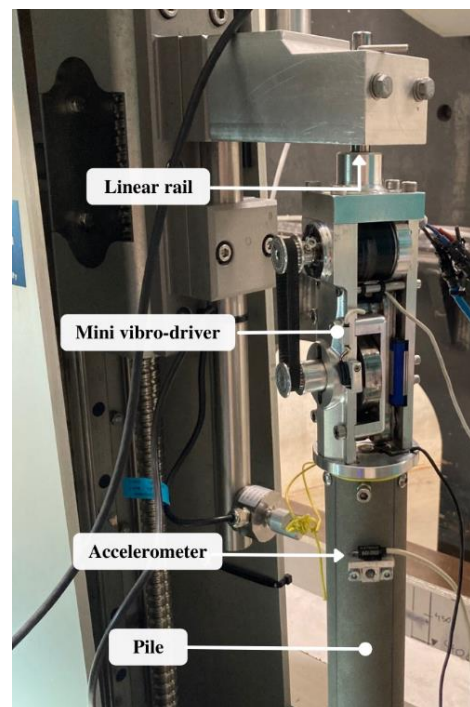


Fig. 2. Setup used for centrifuge vibro-installation test.

3.3 Vibro-driving conditions

Further considerations about the scaling laws adopted to develop the mini vibro-driver and for the preliminary test are explored in Mazutti et al. (2022).

The installation test was performed at a vibrational frequency of 340 ± 10 Hz and with a dynamic force about 1.2 times the self-weight of the system ($F_{\text{dyn}}/F_o \approx 1.2$). In the field, this force ratio must be kept higher than unity so that the dynamic vibrational forces exceed the static forces. The field frequency (of typically around 23 Hz) was not scaled directly in the test (where the frequency required for direct inertial scaling would be $23 \times 30 = 690$ Hz), but was sufficiently high to generate a representative number of cycles during penetration and to cause large enough accelerations in the pile. The frequency of 340 represents a field frequency of 11.3 Hz. This parameter will be explored further in future work.

3.4 Test results

The pile was first lowered into the soil at 1g, penetrating slightly under its self-weight. During spin-up of the centrifuge to 30g, the pile self-weight increased slowly (as well as the stress conditions in the soil sample) and the pile reached a penetration depth of about 0.6D. Once equilibrium was established, the pile was ready for vibro-driving.

Preliminary results of the vibro-installation are given in Figure 3. Figure 3a shows that during 60 seconds (model scale) of vibro-installation, the pile penetrated to a depth of 2.7D. At the start of the installation, the pile net penetration per cycle was about 0.025 mm/cycle (Fig. 3b), but at the end the net penetration was smaller than 0.005 mm per cycle, suggesting that refusal conditions were being approached.

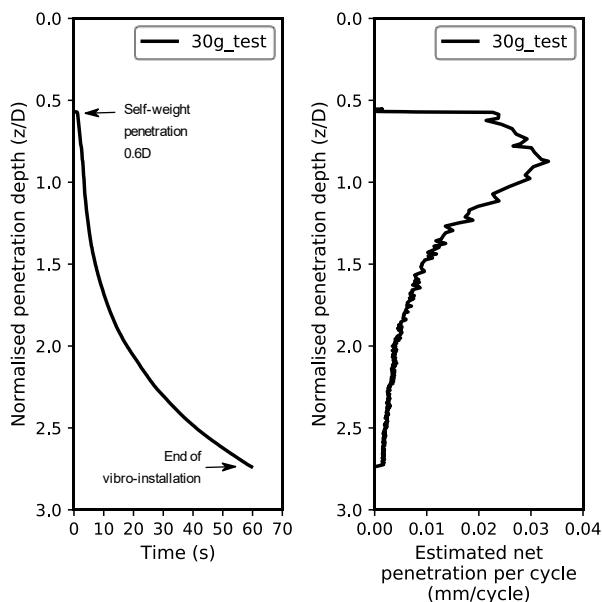


Fig. 3. Vibro-installation test at 30g: (a) normalised penetration depth *versus* time and (b) normalised penetration depth *versus* estimated net penetration per cycle.

The accelerometer measured the amplitude of pile oscillation per cycle due to the dynamic loading with respect to installed penetration depth. The cyclic

amplitude of pile displacement was approximately ± 0.07 mm throughout installation. This agreed closely with the amplitude expected using the known mass eccentricity of the pile driver and the relative masses of the slider mass, vibro-hammer and pile.

The extent of the zone of influence of vibro-installation on the surrounding soil was not directly measured. However, images taken during the test suggest changes at the soil surface occur at distances up to 1.4D away from the pile. This deserves further investigation.

3.5 Discussion of results

The installation of the pile (at least to a depth of 2.7D) in very dense, dry sand conditions with the vibro-driver applying a dynamic force, F_{dyn} of 1.2 times the static weight of the system is encouraging. For this (laboratory) condition, the dry sand ensures that no excess pore pressures can have developed and so is expected to be more onerous than field conditions.

The dynamic pile displacement amplitude of ± 0.07 mm (at model scale) also appears small. However, this corresponds to ± 2.1 mm at prototype scale (approximately typical of field conditions) or $\pm 0.14t$ (so large compared to the pile tip thickness t). Furthermore, approximately 21,000 cycles of loading were applied during the whole installation event, which would be expected to reduce skin friction and likely prevent significant soil plug resistance within the pile. This would be expected to reduce skin friction and likely prevent significant soil plug resistance within the pile. This is also supported by the fact that the cyclic amplitude of the pile did not reduce during the test. At the pile tip, the induced cyclic displacement amplitude appears to be sufficient to unload the pile tip in the upward phase and cause additional penetration per cycle during the downward motion.

4 CONCLUSIONS

This paper has described the challenges of understanding and replicating the soil mechanisms occurring during vibro-installation in sand. To replicate these soil mechanisms, it is important to also represent the stress conditions in field. For this reason, a mini vibro-driver was developed to work in a geotechnical centrifuge.

The mini vibro-driver can be mounted on the top of a model monopile and operate with a range of different vibratory parameters. A preliminary vibro-installation test was successfully carried out at 30g, providing confidence in the design. Furthermore, significant pile penetration was achieved even in dry very dense sand at a relatively small force ratio ($F_{\text{dyn}}/F_o \approx 1.2$); this is encouraging.

The mini vibro-driver opens a wide range of possibilities for the study of vibro-installation. Further tests on pile drivability and pile post-installation

performance are planned.

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