Probability theory applied to pile driveability predictions based on the wave equation

Joël Sinke [i)]

i) Design Leader, Hakkers BV, Oudsas 11, Werkendam.

ABSTRACT

Accurate driveability predictions assist in decision-making on pile installation aspects, like hammer choice. These decisions involve significant risks and costs. This paper is a proof of concept which shows that probability theory can be applied to driveability predictions based on the wave equation model to quantify refusal risk. Ten soil parameters are turned into stochastic variables. The variability of these stochastic variables consists of spatial and transformational variability. The former is caused by soil heterogeneity, whilst the latter is caused by uncertainty in the relationship between measurement and soil model parameter. Spatial variability is incorporated in the one-dimensional profile of each parameter by means of a random field. This random field is a function of the vertical scale of fluctuation which is derived from Cone Penetration Test (CPT) data. The horizontal correlation is assumed to be zero. In practice, there will be horizontal correlation, but this cannot be derived from common soil research. Soil stratification (arrangement of soil layers) is assumed to be deterministic. The transformation variability of the stochastic variables is determined by gathering experimental data from previous research. Based on the variability of the parameters, Monte Carlo simulations are done to quantify the risk of refusal. Case studies show that predictions of the refusal risk give outcomes which are of the same order of magnitude as measurements.

Keywords: driveability, probability theory, soil variability, spatial correlation, Monte Carlo simulation, risk of refusal

1 INTRODUCTION

Successful pile driving is all about reducing soil resistance and so causing soil to fail. A successful driveability prediction depends on an exact estimation of soil resistance capacity. In engineering practice a safe design is achieved by taking a safe, conservative value of the resistance capacity. This conservative value is found by taking into account a safety factor. A more sophisticated approach is to take into account the variability of each parameter by a probabilistic approach.

Driveability predictions provide information on pile installation. For example, the hammer with minimum requirements to drive the pile to sufficient depth. The selection of a hammer has a significant effect on the installation costs of a pile. Typically, a stronger hammer is more expensive, but decreases the risk of refusal. Therefore, a trade-off must be made between risk and costs. By quantifying the risk of refusal, decision-making on hammer selection can be improved. Currently pile driveability prediction methods do not predict the probability of pile refusal. Reliability-based design theory must be applied to driveability predictions to incorporate uncertainty.

Reliability-/risk-based design calls for a willingness to accept that absolute safety is an unattainable goal and that probability theory can provide a formal framework for developing design criteria that would ensure that the probability of failure (used herein to refer to pile refusal) is acceptably small.

This paper, which summarizes previous research (Sinke, 2020), explains how probability theory is incorporated in a driveability prediction method (Allwave-PDP).

2 WAVE EQUATION MODEL

Driveability predictions based on the wave equation (solved by the method of characteristics) has proven to be the most accurate prediction method (Middendorp, 2004). Pile driving is not a problem of impact that may be solved directly by Newton's laws. The driving force is transmitted by waves. This propagation of waves is described by the wave theory. The wave equation is the most important formula in this field.

Pile, hammer and soil form a pile driving system. The (vibro or impact) hammer generates the driving force to drive the pile in the soil. In the model the soil is simplified to one-dimensional properties.

In the TNOWAVE model the continuous soil friction is replaced by a number of concentrated frictional forces. The friction/resistance is modelled by a spring-damper system, see Figure 1. This leads to resistance as function of displacement and velocity. The resistance is both modelled at toe and along the shaft. The physical process at toe and shaft are significantly different due to its different failure mechanisms.

Static resistance of the soil is modelled according to an elasto-plastic spring defined by quake (u_q) and yield stress (F_y), see Figure 2. Damping (velocity-dependent resistance) is modelled as:

$$R_d = Cv^{a} \tag{1}$$

Where C = damping constant [kNs/m] and α = damping exponent [-]. For α = 1 linear (viscous) damping is generated, while for α < 1 a parabolic one is generated.

Due to cyclic loading fatigue occurs in the soil. Fatigue leads to decrease of yield stress of the soil. The ratio of initial to residual yield stress is the β-factor. The β-factor is applied to the static shaft and toe resistance of the soil.

The wave equation model is physically based, i.e. the principles of the physical process are included in the model. This makes this model suitable for a probabilistic approach. Namely, the parameters are measurable in contrast to an empirical model.

In this paper the software program Allwave-PDP is used. This software is widely applied and has proven reliability.

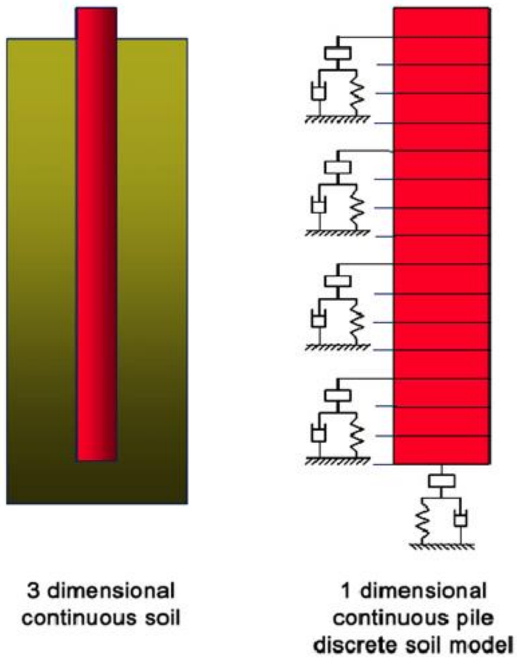

Fig. 1. Driveability model (TNOWAVE) (Middendorp, 2012)

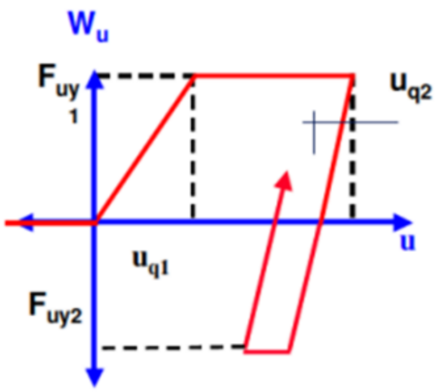

Fig. 2. Spring model (Middendorp, 2012).

3 PROBABILITY THEORY

Probability theory is the framework to analyse random events in a logical way. This framework is applied on the parameters of a driveability model. All parameters of the pile driving system have variability. However, the variability of pile and hammer parameters is in general much smaller than soil parameters, because pile and hammer parameters are easily measurable, whilst soil is a natural product and is not easily measurable. In this probabilistic model hammer and pile properties are assumed as deterministic values, i.e. no randomness/variability is assumed.

Soil variability consists of spatial variability, which is caused by soil heterogeneity, and transformation variability, which is caused by uncertainty in the relationship between measurement and model parameter.

A number of ten soil parameters (both shaft and toe) of the wave equation model are turned from deterministic parameters into stochastic parameters:

- Yield stress
- Quake
- β-factor
- Damping constant
- Damping exponent

A stochastic parameter has a probability distribution. Variability refers to how spread the values in this distribution are. In the next sections will be explained how soil variability is determined and incorporated in the driveability prediction model.

4 SOIL PARAMETER COMPOSITION

The wave equation model simplifies the soil to a one-dimensional profile. For every soil parameter a profile is randomly generated. The profile of each parameter is composed of various components, see steps in Figure 3 for a visualization. The standardized random field and stratification are identical for all parameters. For each parameter a unique residual standard deviation (σ_{res}) and trend lines are determined.

With step 1 and 2 spatial variability is included. A standard normal random field (μ=0, σ=1) is generated as a function of the *scale of fluctuation* (θ), see section 5.2 for further explanation. By substituting σ_{res} in the standard random field a parameter-specific random field is obtained, see also step 1. This random field is added to a trend line, see step 2. The trend line is derived from the soil parameter based on the characteristic CPT.

To include transformation variability each soil layer is multiplied by a factor sampled from a normal distribution with a parameter-specific coefficient of variation (COV), see step 3 and section 6 for further explanation.

The result is a randomly generated one-dimensional profile of a soil parameter. As a result spatial variability random scatter is added to the trend lines. The transformational variability is accounted for by shifting the profile per layer.

5 SPATIAL VARIABILITY

Geotechnical properties vary both in vertical and horizontal direction. This variability is caused by heterogeneity of the soil (Hicks, 2015). In the Netherlands geotechnical properties are primarily determined by Cone Penetration Test (CPT) research. This results in vertical profiles of cone and shaft resistance. With these parameters the soil type can be determined.

A characteristic CPT must be selected for the soil on which the probabilistic prediction is made. Namely, stratification and trend lines are derived from the characteristic CPT, see also Figure 4 for an example. In Figure 4 the CPT consists of 3 layers. The trend lines of each layer are shown in red.

Typically, a soil consists of various geotechnical layers. These layers result in abrupt property changes at their boundaries. Within layers the geotechnical properties are relatively homogeneous; variability is mostly limited to cone resistance, whilst soil type does not vary. The properties of the layers (thickness and depth) can vary greatly in horizontal direction. A CPT gives limited insight in the variability in the horizontal direction. The horizontal variability can be qualitatively observed when multiple CPT's of a certain area are available.

5.1 Scale of fluctuation

A CPT gives a substantial amount of data points (every 0.02 m) in a vertical line. This makes a CPT suitable to extract the vertical scale of fluctuation of CPT properties. The scale of fluctuation defines the distance beyond which there is no significant correlation and is used to quantify spatial variability.

To make a CPT suitable for extraction of the scale of fluctuation, the data must be detrended and normalized (Lloret-Cabot, 2014). Detrending implies removing the linear depth trend from the data, see figure 4. Consequently, the detrended data is divided by its residual standard deviation (σ_{res}) to normalize the data. The result is a standard normal field (μ=0, σ=1). Note that this process is an inverse process of the profile composition. In the composition the field is multiplied by σ_{res} and the trend is added.

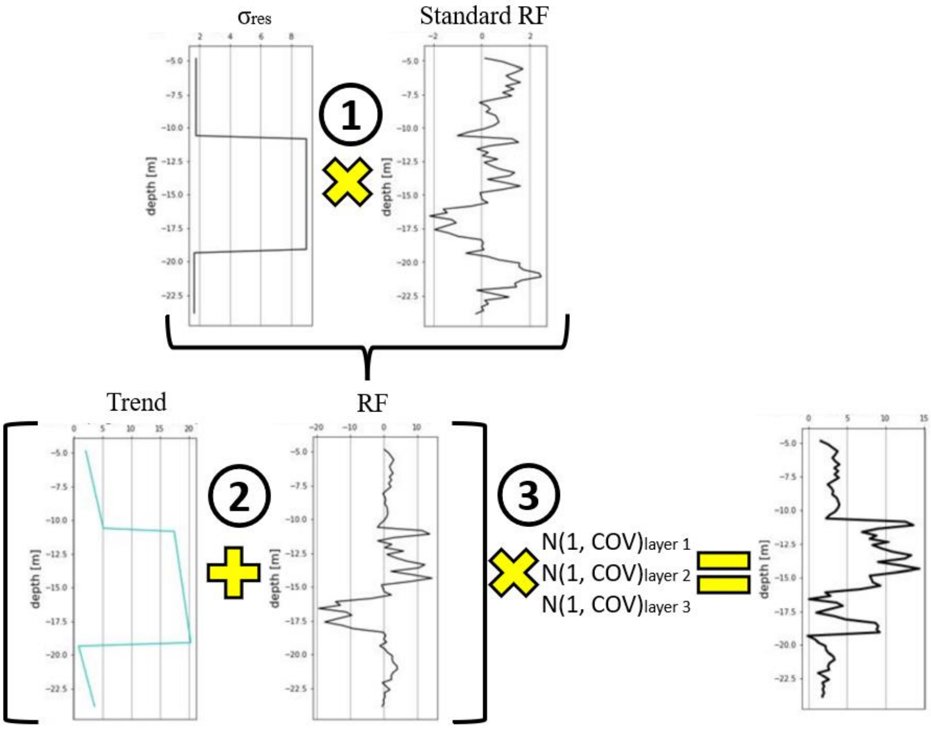

Fig.3. Composition of one-dimensional soil parameter.

The scale of fluctuation is extracted from the standard normal field by fitting a theoretical and experimental autocorrelation function.

The (experimental) autocorrelation function follows from analysis of the standard normal field. This function is fitted to a theoretical function. This is done by fitting the scale of fluctuation, which is a parameter of the theoretical function. It is assumed that the theoretical function has an exponential shape (Lloret-Cabot, 2014). This implies that with increasing distance between two points, the correlation decreases exponentially.

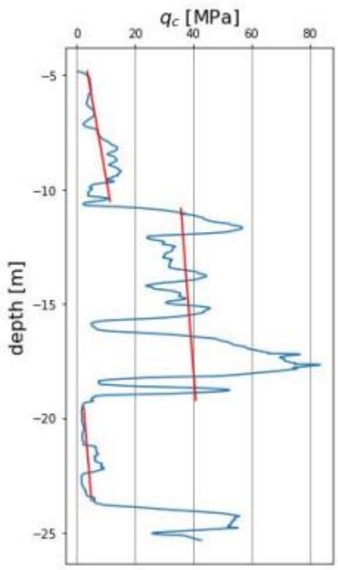

Fig. 4. CPT (cone resistance) including trend lines per layer

5.2 Soil stratification

In the probabilistic model the soil stratification (arrangement of layers) is considered as deterministic. Each layer is considered as an independent soil profile, see for example Figure 4 where 3 layers are identified. Due to the independence of the layers there is no correlation between its properties. This is a simplification to make a workable model. In some CPT's no clear stratification can be distinguished. This requires engineering judgement. In reality, there will always be variability in the stratification in the horizontal plane, e.g. depth of a layer boundary will vary and in non-uniform soils layers might even disappear from one CPT to the next.

The more the stratification varies, the more limited the valid distance from the CPT is.

5.3 Horizontal variability

In a typical Dutch foundation project (e.g. a quay wall) the horizontal distance between CPT's is 25-50m. Horizontal correlation cannot be accurately estimated with this distance. In the probabilistic model the horizontal correlation is assumed zero when the one-dimensional random field is generated. Implicitly, some horizontal correlation is incorporated in the random field as the stratification and trend lines are considered deterministic.

This approach leads to conservative results for predictions in 10-15 m proximity of the selected CPT. Namely, in this proximity horizontal correlation is significant and it is therefore expected that in reality the spatial variation is small nearby the CPT.

6 TRANSFORMATION VARIABILITY

A transformation model relates a field measurement (e.g. cone resistance) to a model parameter (e.g. toe damping). These relationships are found by laboratory experiments or matching of predictions and practice (postdictions). Usually the best estimate (as a single value) of these experiments and postdictions is used in prediction models. In a probabilistic model the variability of model parameters is considered.

CPT data only measures static parameters. The dynamic parameters must be derived from the CPT data by establishing a relationship between static parameters and dynamic parameters.

Transformation variability is applied to the soil parameters as mentioned in section 3. Except the yield stress, because this parameter is directly measured by the CPT. Therefore, there is no transformation variability.

In Table 1 the coefficient of variation (COV), which is the ratio of the standard deviation to the mean, is shown for each parameter. This statistical property shows the variability in relation to the mean and is a dimensionless number. This makes the value independent of the measurement unit. Hence, also measurement data in other units can be used to determine variability. The sections below explain how the transformational variability is established.

Table 1. Coefficient of variation for stochastic variables.

Soil parameter	Shaft	Toe
Quake	41%	10%
β-factor	32-64%	32-64%
Damping constant	Sand/clay: 32/34%	58%
Damping exponent	19%	19%

6.1 Quake

Research data is available on shaft quake values of displacement piles (McVay and Kuo, 1999). Shaft friction is largely the same phenomenon in displacement and non-displacement piles.

For toe quake no experimental data is available. The Young's modulus is to a certain degree analogous to quake as both parameters define a relationship between stress and strain. Therefore, the variation according to NEN9997-1 soil Young's modulus is adopted for toe quake.

6.2 β-factor

Analysis on β-factor in a collection of case studies on sandy soils (FR<1,0) leads to a COV of 0.32 (Robertson, 2010). As no conclusive research on β-factor of clay can be found, it is assumed that the COV for clay is double that of sand. Few systematic research has been carried out on fatigue due to cyclic loading on clay. Clay behavior is more complex than that of sand, because of its dependency on such factors as time-dependent creep and preconsolidation periods which can be overlooked for sand (Yasuhara, 1992).

6.3 Damping constant

One test set-up measured the toe damping of clay and sand by a falling load on a sample (Coyle & Gibson, 1970). A falling load resembles the impact of the pile toe on the soil. In another experiment toe and shaft damping of clay samples is measured by pushing a rod in a sample at different speeds (Lithouki & Poskitt, 1980). Due to a lack of experiments on shaft damping of sand, the variability of clay is assumed to be valid for sand.

6.4 Damping exponent

The range of 0.17 – 0.37 found in literature for the damping exponent is assumed as a 95% confidence interval (Lee, 1988).

7 MONTE CARLO SIMULATION

In a Monte Carlo (MC) simulation a process is repeated a number of times in which the input parameters vary according to probability distributions. This MC simulation is done for a driveability prediction in which the soil parameters are randomly generated as described in the previous sections. The result is a number of penetration speed-depth-curves (v_p-z-curves), see Figure 5. The probability of refusal is defined as the proportion of the total number of simulations in which refusal occurs (defined as non-exceedance of a penetration speed of 10 mm/s, see vertical red line). Three case studies have been done to prove that this probabilistic method gives realistic results.

7.1 Case studies

All cases are foundation project in which sheet piles are driven with vibro hammers. Elaborate project data is available, like geotechnical reports and data on hammer and sheet pile type. In Table 2 a comparison on prediction and measurement is given.

Table 2. Results prediction and measurement of refusal rate

Case	Prediction	Measurement
Den Oever	33%	50%
Woudsend	18%	25%
Rotterdam	0%	0%

These results show that the prediction results have the same order of magnitude as the measurements. Please note that for the Rotterdam case study no driveability issues occurred. The MC simulation was made to check if the model does not present false positives (i.e. predicting refusal where driving goes well).

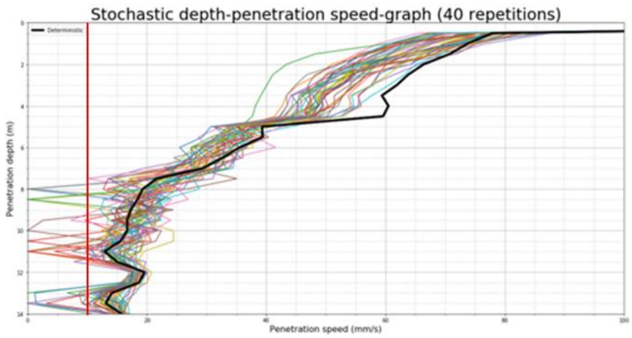

Fig. 5. Visualization of MC simulation in v_p-z-graph.

8 CONCLUSIONS

The probabilistic driveability prediction gives extra information compared to the usual prediction, namely the risk of refusal. This is useful when pile installation aspects are considered. A MC simulation is an intuitive method to take into account the variability of the pile driving system parameters. Case studies show that model is a proof-of-concept of application of probability theory to driveability predictions.

Only the variability of soil parameters is considered. The other pile driving system components; pile and hammer are assumed as deterministic parameters in this model. The soil variability is split into transformational variability and spatial variability. The former is caused by an uncertain relationship between measurement (CPT) and model parameters. The latter is caused by soil heterogeneity and takes into account that a CPT is just a local representation of the soil. A random field based on the vertical scale of fluctuation of the CPT is added to the trend line to model the spatial variability. The transformation variability is modelled by multiplying each soil layer by a randomly generated factor.

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